EXPERIMENTAL PERFORMANCE EVALUATION OF A 6.02-INCH RADIAL-INFLOW TURBINE OVER A RANGE OF REYNOLDS NUMBER

by Donald E. Holeski and Samuel M. Futral, Jr.

Lewis Research Center
Cleveland, Ohio

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • JANUARY 1967

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SUMMARY

An experimental performance evaluation of a 6.02-inch-tip-diameter radial-inflow turbine using argon as the working fluid was made over a range of inlet total pressure from 1.2 to 9.4 pounds per square inch absolute with corresponding Reynolds numbers from 20 000 to 225 000. Reynolds number, as used herein, is defined as the ratio of the weight flow to the product of viscosity and rotor tip radius, where the viscosity is determined at the turbine-entrance condition. Efficiency and equivalent weight flow increasing with increasing inlet pressure and Reynolds number. At design equivalent speed and pressure ratio, the total efficiency increased from 0.85 to 0.90, while the static efficiency increased from 0.80 to 0.84 with increasing Reynolds number. The corresponding increase in equivalent weight flow was about 2 percent. The relation observed between experimentally determined efficiency and Reynolds number indicated that approximately 70 percent of the turbine losses are associated with wall and blade boundary layers.

A study was made at design Reynolds number for determining the probable error of a single observation for measured variables and calculated quantities. The results from a 16 data point set revealed that the probable errors in total and static efficiencies were ±0.009 and ±0.008, respectively. The results were extended over the Reynolds number range and showed that an increase in probable error occurs as Reynolds number decreases.

INTRODUCTION

Experimental and analytical studies of Brayton cycle space power systems are currently being conducted at Lewis Research Center. The space power systems of

interest cover a wide range of power levels varying from a few kilowatts to several hundred kilowatts. At the present time, experimentation is being conducted on components of 10-kilowatt-shaft output systems. Interest has developed in knowing the performance of a given turbine operating over a power-level range. Since turbine Reynolds number is proportional to power level, it is then important that the effect of this parameter on the turbine performance be understood.

Considerable effort has already been directed toward studying the effect of Reynolds number for compressors and axial-flow turbines. Little information is available, however, on the effect of a change in Reynolds numbers on the performance of radial-inflow turbines. An investigation that covered a range of Reynolds number from 230 000 to 590 000 showed a decrease in efficiency with decreasing Reynolds number (ref. 1).

This report describes the results of an experimental investigation of the performance of a radial-inflow turbine over a Reynolds number range from 20 000 to 225 000. The turbine used in the present investigation was a 6.02-inch-tip-diameter radial-inflow unit, described in reference 2 where only a limited Reynolds number range (64 000 and 224 000) was covered. This turbine was designed as the compressor-drive unit for a two-shaft 10-kilowatt-shaft output system (ref. 3) that would use solar energy as the heat source and argon as the working fluid.

The tests were conducted at constant design equivalent speed at a series of fixed inlet pressures that ranged from 1.2 to 9.4 pounds per square inch absolute and at inlet temperatures that ranged from 540° to 670° R. The turbine exhaust pressure was varied over a range at each inlet pressure. The results of these tests in terms of efficiency, weight flow, and torque are presented. In addition, since measurement and control problems are more difficult at low levels of pressure and flow, a study of probable error was made, based on scatter data.

TURBINE DESCRIPTION

The turbine used in this investigation was a 6.02-inch-tip-diameter radial-inflow turbine designed for a 10-kilowatt-shaft output space power system with argon as the working fluid. This particular space power system was designed for two-shaft operation with a high-speed turbocompressor and a low-speed turboalternator. The compressor drive turbine, which is the turbine investigated herein, has the following design values (see appendix A for symbols):

Total efficiency, η_T 0.880
Static efficiency, η_s 0.824
Total- to total-pressure ratio, p_2'/p_3' 1.560

Total- to static-pressure ratio, p_2'/p_3 . . . . 1.613
Turbine speed, N, rpm 38 500
Specific work, Δh, Btu/lb 34.73
Argon weight flow, w, lb/sec 0.611
Inlet total temperature, T_2', ^oR 1950
Specific speed, N_s 95.6
Inlet total pressure, p_2', psia 13.20
Blade-jet speed ratio, ν 0.697
Reynolds number, Re 63 700

The air-equivalent (U.S. standard sea level) design values were as follows:

Equivalent total- to total-pressure ratio, p_2'/p_3' 1.496
Equivalent total- to static-pressure ratio, p_2'/p_3 . . . . 1.540
Equivalent turbine speed, $N\sqrt{\theta_{cr}}$, rpm 22 527
Equivalent specific work, $\Delta h/\theta_{cr}$, Btu/lb 11.9
Equivalent weight flow, $\left(w\sqrt{\theta_{cr}}/\delta\right)\epsilon$, lb/sec 1.063
Blade-jet speed ratio, ν 0.697
Equivalent torque, $(\Gamma/\delta)\epsilon$, in.-lb 50.05

Figure 1. - Turbine rotor and scroll.

84°31'

16°21'49"

1.812"

11 Blades
equally spaced

11 Splitter blades
equally spaced

t

2.147"   0.750"   3.075"

3.010"   3.787"

r

14 Stator blades equally spaced
(The blades are identical except
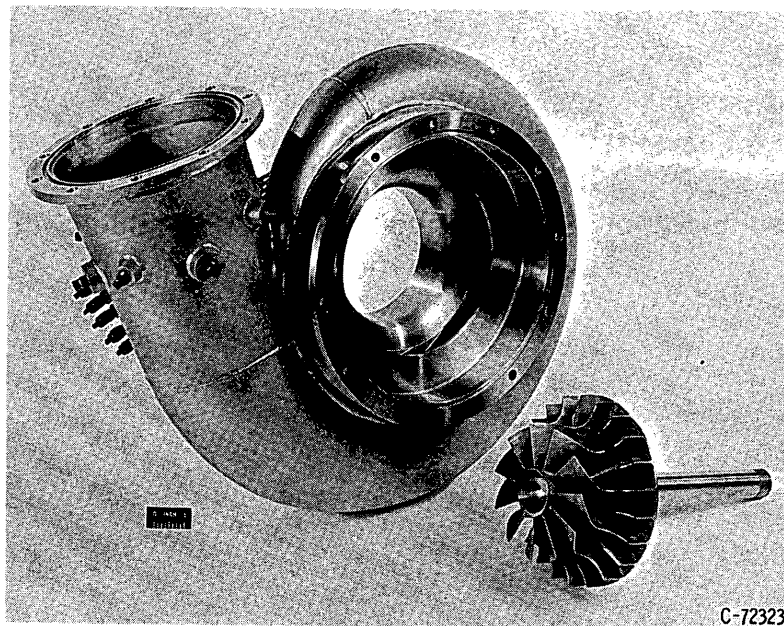for one blade that is extended to
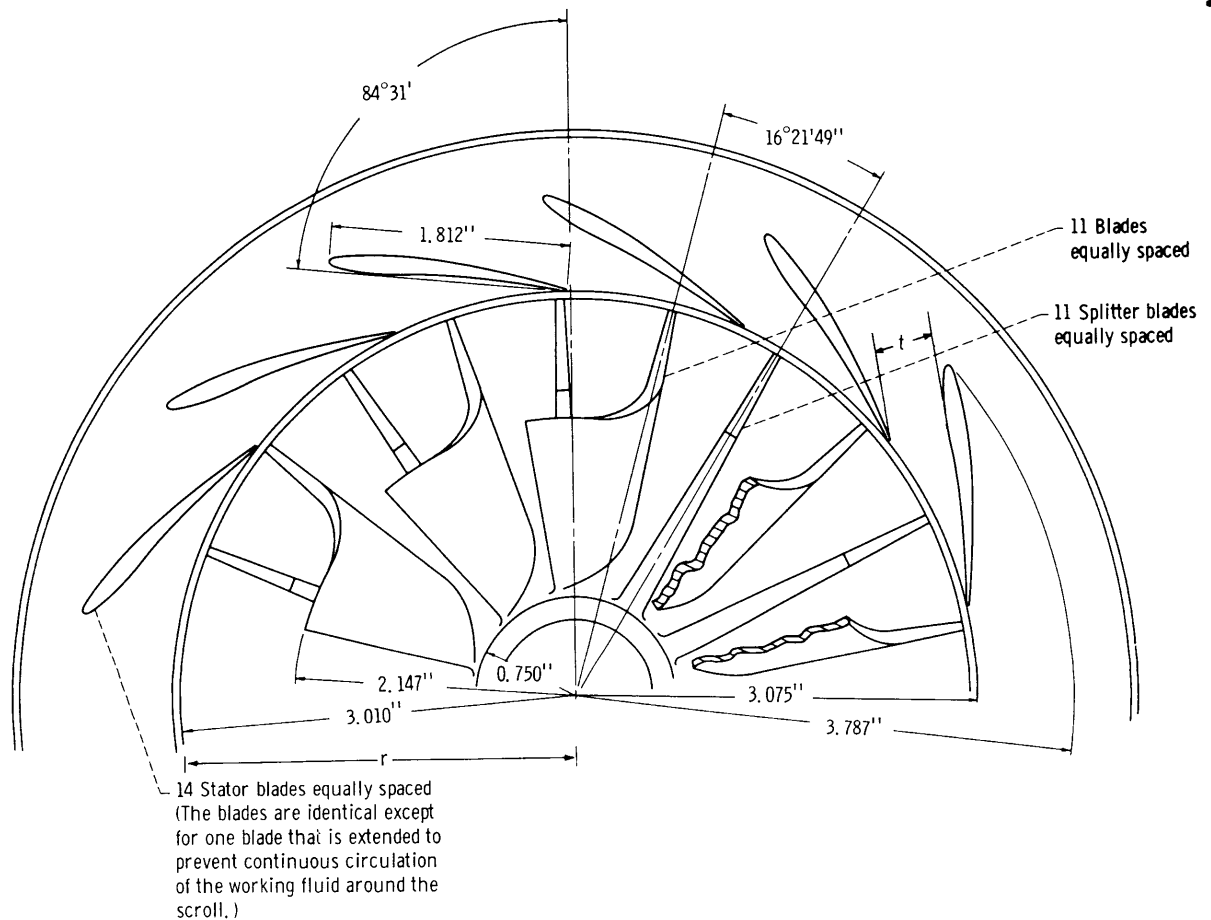prevent continuous circulation
of the working fluid around the
scroll.)

Figure 2. - Schematic drawing of turbine stator and rotor.

A detailed description of the turbine investigated is given in reference 2, but for convenience, the significant features are discussed here. Also, a photograph of the turbine is shown in figure 1. The stator and rotor blades, along with their major dimensions, are presented in figure 2. The stator consisted of 14 blades, equally spaced. The blades were identical except for one blade, which was extended for the purpose of preventing continuous circulation of the working fluid around the scroll. The rotor had 11 full blades and 11 splitter blades. The splitter blades are used over the initial third of the rotor, as shown in figure 1. Flow throughout the turbine was subsonic.

APPARATUS, INSTRUMENTATION, AND PROCEDURE

The apparatus, instrumentation, and procedure used in determining the performance of the turbine over the range of Reynolds number are similar to those reported in refer-
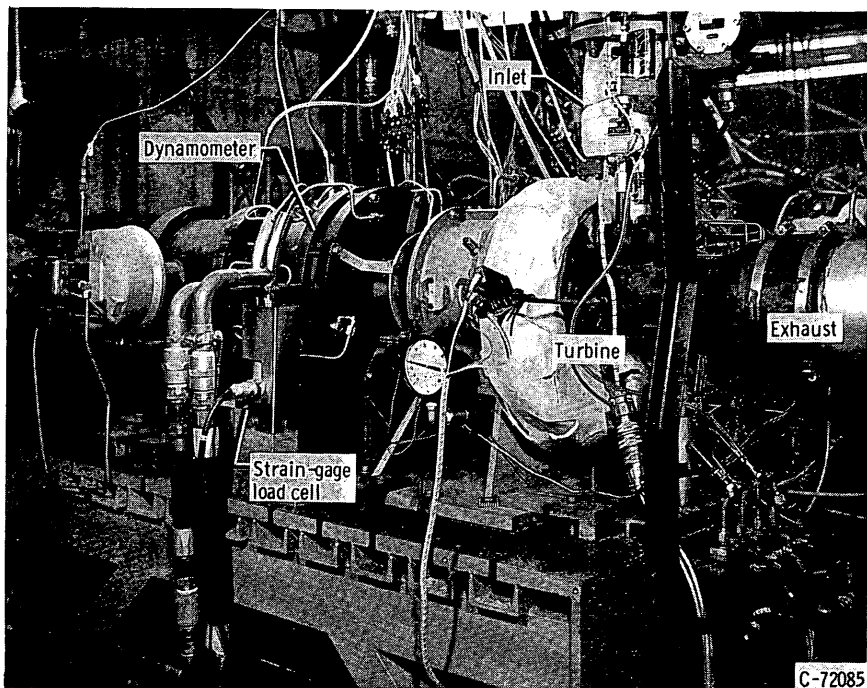
4

Figure 3. - Turbine test facility.

ence 2. The arrangement of the apparatus was the same, and a photograph of the turbine test facility is shown in figure 3. For tests described herein, only argon was used as the working fluid.

The instrumentation is also similar to that previously reported (ref. 2), and the station locations are shown in figure 4. Manometer tubes were installed in addition to the electrical transducers to measure the turbine pressures. For pressures of 3.4 pounds per square inch absolute and lower, the manometers were used to measure all turbine pressures. These manometers contained a fluid with a specific gravity of 1.04 and were used as absolute manometers where the reference side of each manometer was evacuated to a few microns of mercury. All other data were recorded by an automatic digital potentiometer and all data were processed through an electronic digital computer.

As mentioned in the INTRODUCTION, experimental performance data of the turbine were obtained in reference 2 at Reynolds numbers of 64 000 and 225 000. These Reynolds numbers correspond to inlet total pressures of 3.4 and 9.4 pounds per square inch absolute, respectively. For the subject investigation, additional data were taken at pressure levels of 1.20, 1.65, 2.26, and 4.74 pounds per square inch absolute. Values of Reynolds number for each level were determined with the turbine operating at a design blade-jet speed ratio of 0.697. These values are 20 000, 27 000, 43 000, and 103 000, respectively, and correspond to the previously mentioned inlet total pressures. A range of inlet-total- to exit-static-pressure ratios from about 1.4 to 1.8 was covered at each level of inlet total pressure. The speed was held constant at design equivalent
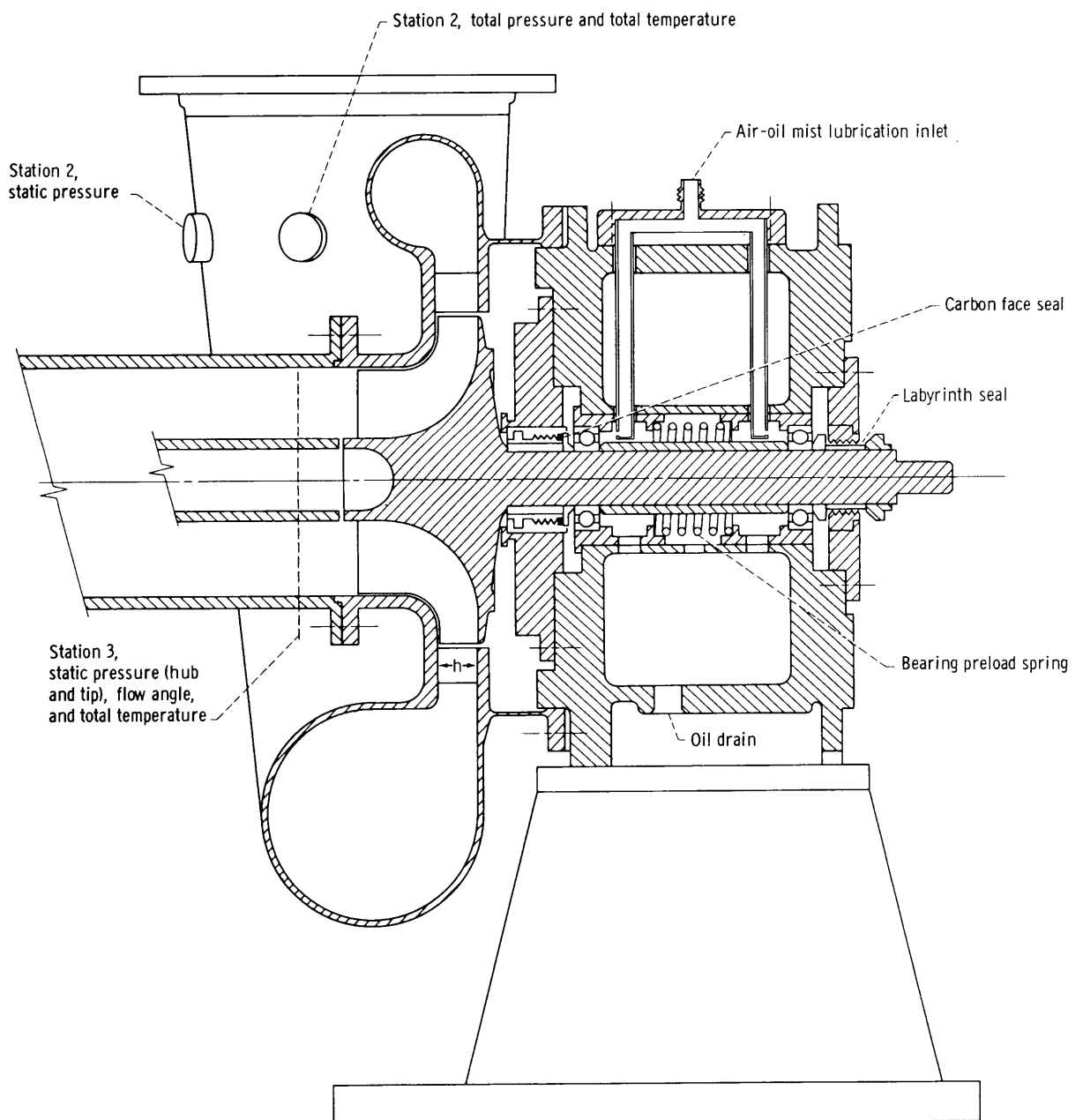
5

Station 2, total pressure and total temperature

Air-oil mist lubrication inlet

Station 2, static pressure

Carbon face seal

Labyrinth seal

Station 3, static pressure (hub and tip), flow angle, and total temperature

h

Bearing preload spring

Oil drain

Figure 4. - Turbine showing instrumentation stations (ref. 2).

TABLE I. - INLET CONDITIONS
CORRESPONDING TO REYNOLDS
NUMBERS INVESTIGATED

Reynolds number, Re	Inlet total pressure, psia	Inlet total temperature, °R
[a]225 000	[a]9.4	[a]540
103 000	4.74	540
[a]64 000	[a]3.4	[a]620
43 000	2.26	620
27 000	1.65	670
20 000	1.20	660

[a]Obtained from ref. 2.

value for all inlet total pressure levels. Since the value of turbine Reynolds number is dependent on inlet total temperature and pressure, these quantities were varied to obtain the desired value of turbine Reynolds number.

The values of inlet total temperature and pressure along with corresponding Reynolds numbers at design equivalent speed and pressure ratio are shown in table I. Also included are points at inlet total pressures of 3.4 and 9.4 pounds per square inch absolute, corresponding to Reynolds numbers of 64 000 and 225 000, which were taken from reference 2 where the data were also taken at design equivalent speed and pressure ratio. The performance calculations were made in the same manner as reported in reference 2.

TURBINE REYNOLDS NUMBER

In turbomachinery work, Reynolds number is calculated in several ways. In this report the expression used is

$$Re = \frac{w}{\mu r_t}$$

(All symbols are defined in appendix A.) This expression may be obtained from the conventional expression

$$Reynolds\ number = \frac{\rho V h}{\mu}$$

where the blade height h is used as the characteristic length and represents the passage height at the stator exit. The weight flow leaving the stator is

$$w = \rho A V_r = (\rho)(2\pi r_t h)(V \cos \alpha)$$

where α is the exit angle of the stator, and A is the flow area. Rearranging the expression for the weight flow gives

$$\rho V = \frac{w}{2\pi r_t h \cos \alpha}$$

and substituting into the previous equation for Reynolds number, the expression obtained is

$$\text{Reynolds number} = \frac{1}{2\pi \cos \alpha} \frac{w}{\mu r_t}$$

It is convenient to drop the factor $1/(2\pi \cos \alpha)$, which leaves the expression used in this report

$$\text{Re} = \frac{w}{\mu r_t}$$

The viscosity is evaluated at the turbine-entrance condition. In appendix B, this form is compared with certain others that are in common use.

RESULTS AND DISCUSSION

The results of this investigation are presented in three sections. The first section presents the performance characteristics of the turbine at six different inlet total pressures, and thus at six different Reynolds numbers. The second section shows the calculated overall loss variation of the turbine over the range of Reynolds number. The third section discusses the probable error of efficiency and weight flow for a set of data points at a single Reynolds number and includes a discussion of the extension of probable error results over the Reynolds number range.

Performance Over Range of Inlet Total Pressure

The performance results of the subject turbine over a range of inlet total pressures from 1.2 to 9.4 pounds per square inch absolute are shown in figures 5(a) to (f) in terms of static efficiency and the total efficiency plotted against blade-jet speed ratio. These figures show that a decrease in efficiency occurred as the inlet total pressure was decreased. The general shape of all the curves is similar, and no large or abrupt change in performance occurred over the range of inlet pressures investigated. For each of the
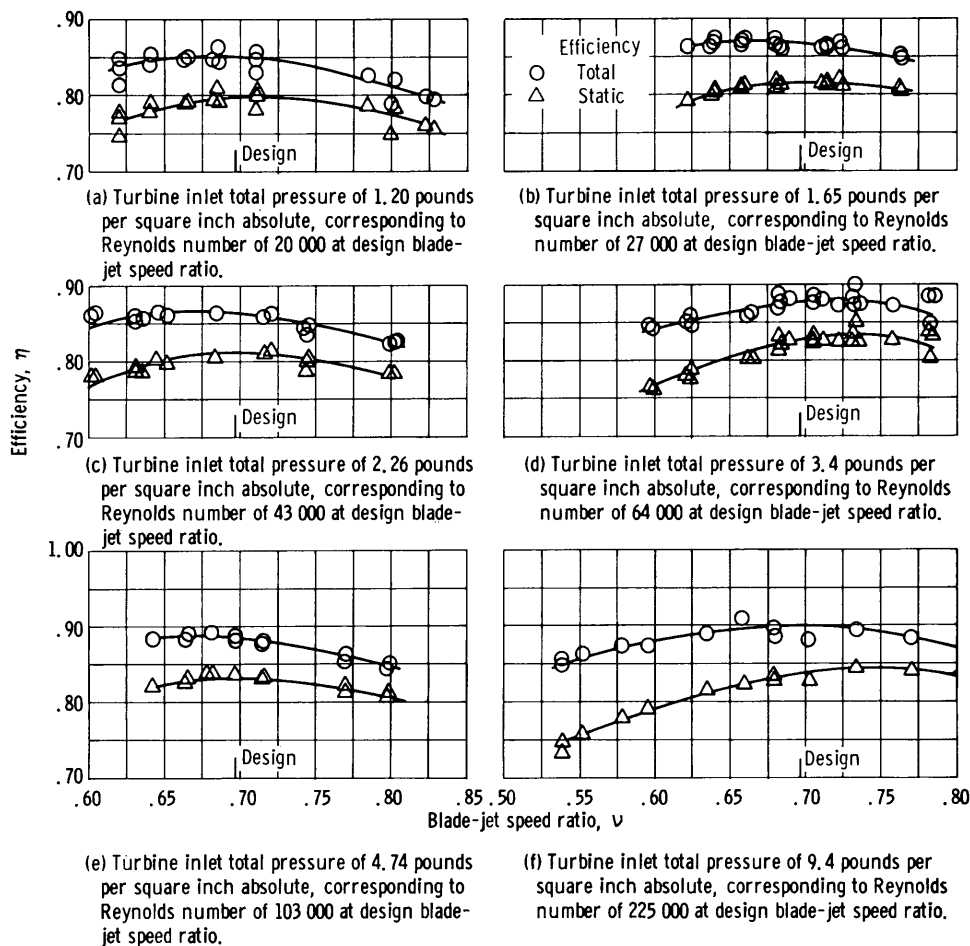
(a) Turbine inlet total pressure of 1.20 pounds per square inch absolute, corresponding to Reynolds number of 20 000 at design blade-jet speed ratio.

(b) Turbine inlet total pressure of 1.65 pounds per square inch absolute, corresponding to Reynolds number of 27 000 at design blade-jet speed ratio.

(c) Turbine inlet total pressure of 2.26 pounds per square inch absolute, corresponding to Reynolds number of 43 000 at design blade-jet speed ratio.

(d) Turbine inlet total pressure of 3.4 pounds per square inch absolute, corresponding to Reynolds number of 64 000 at design blade-jet speed ratio.

(e) Turbine inlet total pressure of 4.74 pounds per square inch absolute, corresponding to Reynolds number of 103 000 at design blade-jet speed ratio.

(f) Turbine inlet total pressure of 9.4 pounds per square inch absolute, corresponding to Reynolds number of 225 000 at design blade-jet speed ratio.

Figure 5. – Performance characteristics over range of turbine inlet total pressure.

figures, (5(a) to (f)), the Reynolds number value is shown for the design equivalent speed and pressure ratio.

The values of efficiencies are plotted against Reynolds number in figure 6. The efficiency values were taken from figures 5(a) to (f) at design blade-jet speed ratio for these curves. A decrease in performance as the Reynolds number is reduced from a high value is clearly shown in figure 6. The total efficiency decreases from about 0.90 to 0.85, and the static efficiency decreases from about 0.84 to 0.80. This decrease in efficiency with decreasing Reynolds numbers is gradual, as no abrupt changes occurred.

The variation of equivalent weight flow with Reynolds number is presented in figure 7. The values of weight flow used in this figure are at design blade-jet speed ratio and were obtained from faired curves of equivalent weight flow against blade-jet speed ratio at each of the inlet total pressures investigated. This figure shows a decrease in equivalent weight flow over the Reynolds number range. The decrease is approximately 2 percent for the low Reynolds number data as compared with the high Reynolds number data.
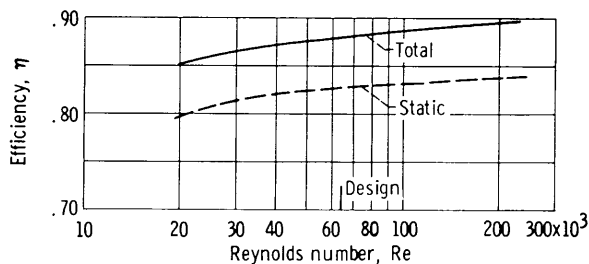
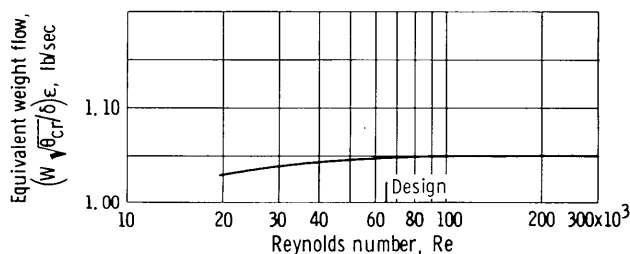Figure 6. - Effect of Reynolds number on turbine efficiency at design blade-jet speed ratio.

Figure 7. - Variation of equivalent weight flow with Reynolds number at design blade-jet speed ratio.
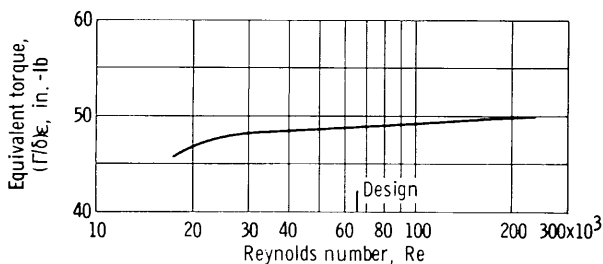

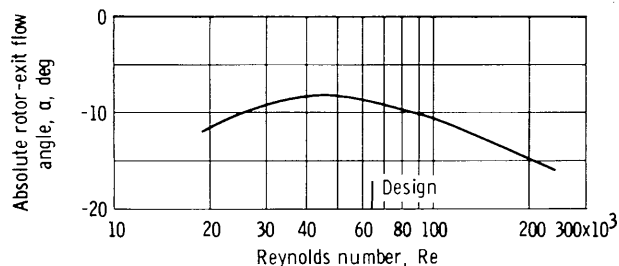Figure 8. - Variation of equivalent torque with Reynolds number at design blade-jet speed ratio.

Figure 9. - Variation of absolute rotor-exit flow angle at mean radius with Reynolds number at design blade-jet speed ratio.

The variation of equivalent torque with Reynolds number is presented as being indicative of the output power of the turbine (fig. 8). As in the preceding figures, the curve shown is for design equivalent speed and pressure ratio. This figure shows a decrease in equivalent torque as Reynolds number decreases. The lowest value of equivalent torque is approximately 6 percent lower than that obtained at the highest Reynolds number. Some of this decrease is a result of the equivalent weight flow decreasing over the Reynolds number range; the remainder results from the decrease in efficiency.

A plot of the absolute gas angle at the turbine exit over the range of Reynolds number investigated is shown in figure 9. The measurements were made at the mean radius. Negative values of the angle indicate that the tangential velocity component has a direction opposite to that of the blade velocity. The curve shows the exit gas angle to be negative over the entire range and to vary from -12° at a Reynolds number of 20 000 to -16° at a Reynolds number of 225 000. At a Reynolds number of 43 000, the exit gas angle is shown to be nearest axial at -8°. This angle change can be considered small, and the variation could result from the combination of losses as they vary with Reynolds number.

Observation of the stator-exit static pressure shows that little variation of pressure ratio occurred across the stator over the Reynolds number range. This indicates that there was little change in reaction through the turbine over the

10

range of Reynolds number investigated; thus, reaction was not a factor in the performance change that was observed for this turbine.

Overall Turbine Losses

Several equations have been used in the past in attempts to relate turbine efficiency to Reynolds number conveniently. A discussion of such equations is included in reference 4. One form of these equations expresses the turbine losses as being partly a result of skin friction and partly a result of kinetic losses, where only skin friction losses vary with Reynolds number. The skin friction losses are always assumed to vary with the 1/5 power of the Reynolds number ratio. This approach appears to be better than the simple exponential type of equation. If the assumption is made that 70 percent of the turbine losses are associated with skin friction (i.e., wall and blade boundary-layer losses) and 30 percent with kinetic losses, the equation has the form

$$\frac{1 - \eta}{1 - \eta_{ref}} = 0.3 + 0.7 \left(\frac{Re}{Re_{ref}}\right)^{-1/5}$$

It should be realized that the percentage of turbine losses related to skin friction could differ with different types of turbines.

Examination of the experimental data was made and the results are presented in figure 10. The faired data of total efficiency at design blade-jet speed ratio from figure 6 were used. At the Reynolds numbers investigated, the experimental data are in good agreement with the preceding equation. Thus, the assumption that 70 percent of turbine losses are associated with skin friction and that only these vary with Reynolds number is a good approximation for estimating the overall turbine loss variation with Reynolds number.
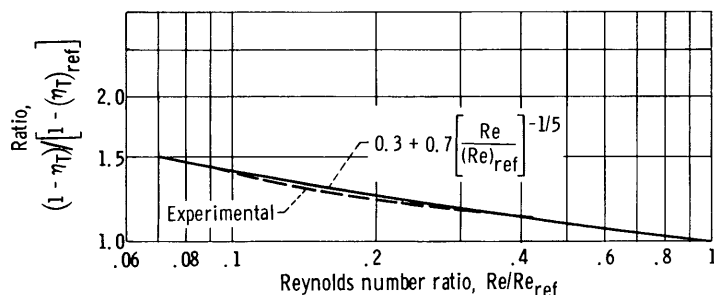

Figure 10. - Variation of experimental turbine losses with Reynolds number ratio.

Error Analysis

In addition to problems ordinarily encountered in the testing of turbomachinery, tests at low power levels give rise to some additional complications. The low flows involved make maintaining steady flow conditions more difficult than operation near design conditions. Since the small pressure differences involved make the use of manometers necessary, instrument lags become significant. Also, the small differences in magnitude of the quantities of interest may approach the resolving ability of the equipment. Because of these additional uncertainties, an examination of the errors involved was advisable. The study is based on scatter, since every effort has been made to calibrate out the systematic errors.

The basic set of data used was a 16 point set taken at essentially design Reynolds number. These readings were used to calculate the probable error in a single observation for each of the measured quantities, consisting of pressures, temperatures, speed, and torque. The method used is described in appendix C, and table II shows both the values of the probable errors calculated and the mean values of the measured variables for the 16-point set of data. Using the probable errors thus obtained for measured quantities, the corresponding probable errors in efficiency, weight flow, and other calculated quantities of interest were calculated at design Reynolds number in the manner explained in appendix C. The results are given in table III. The probable errors of total and static efficiencies were 0.009 and 0.008, respectively.

The probable errors in single observations for the measured variables were also used to extend the results for efficiency and weight flow over the Reynolds number range. A set of six data points was used (with Reynolds numbers of 20 000, 28 000, 43 000, 63 000, 103 000, and 225 000) at design equivalent speed and pressure ratio. For these points, probable errors in the efficiency and weight flow were calculated, as explained in appendix C, by making use of the same absolute values of probable errors previously calculated for measured

TABLE II. - PROBABLE ERRORS FOR EACH

OBSERVATION OF MEASURED VARIABLES

Measured variable	Mean value of variable	Probable error in a single value
Orifice inlet temperature, T_1', °R	631.1	0.23
Orifice inlet pressure, p_1, psia	118.8	0.96
Orifice pressure drop, Δp, psid	3.80	0.041
Rotative turbine speed, N, rpm	21 600	35
Torque, Γ, in.-lb	12.77	0.057
Turbine inlet pressure, p_2, psia	3.38	0.0064
Turbine inlet temperature, T_2', °R	616.8	0.80
Turbine exit hub pressure, $p_{3,h}$ psia	2.05	0.0044
Turbine exit tip pressure, $p_{3,t}$, psia	2.06	0.0032
Turbine exit temperature, T_3', °R	526.1	0.72

TABLE III. - PROBABLE ERRORS FOR
QUANTITIES CALCULATED FROM EACH
SET OF MEASURED VARIABLES

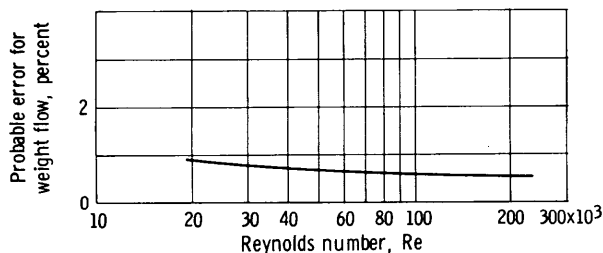| Calculated variable	Value of variable	Probable error, percent of variable
Weight flow, w, lb/sec	0.282	0.76
Actual work, Btu/lb	11.55	0.91
Ideal work for static efficiency, Btu/lb	13.94	0.43
Ideal work for total efficiency, Btu/lb	13.00	0.48
Static efficiency, η_s	0.828	1.01
Total efficiency, η_T	0.888	1.03
Equivalent weight flow, w_{eq}, lb/sec	1.18	0.78

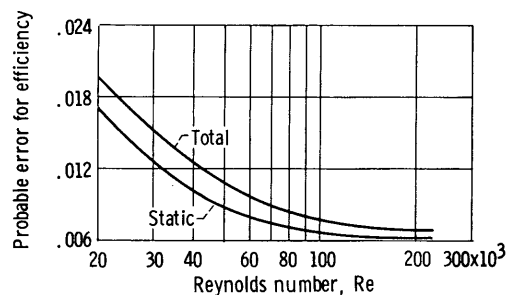Figure 11. - Probable error for a single observation of weight flow.

Figure 12. - Probable error for a single determination of efficiency.

variables at design Reynolds number. These errors are plotted in figures 11 and 12. (It should be mentioned that the effect of scatter is reduced somewhat in curves of final results, such as the efficiency curves of fig. 6, because these curves were cross-plotted from faired curves of fig. 5.) Figure 11 shows that at a Reynolds number of 225 000 the probable error in the weight-flow measurement is about 1/2 percent of the weight flow at that Reynolds number. At a Reynolds number of 20 000, the probable error increased to approximately 1 percent of the weight flow at that Reynolds number. The probable error in the efficiencies shows the same trend of increasing probable error with decreasing Reynolds numbers. Figure 12 presents the values of probable error of each calculated value of efficiency over the Reynolds number range. For a Reynolds number of 225 000, the probable error in total efficiency is about 0.007, and at a low Reynolds number of 20 000, the probable error increased to approximately 0.02. The probable error of static efficiency shows a similar curve, where, at a Reynolds number of 225 000, the probable error was 0.006 and at a Reynolds number of 20 000, the probable error increased to 0.017.

It should again be noted that each curve drawn through a set of calculated points has an averaging effect and, consequently, an accuracy greater than that of the individual points.

SUMMARY OF RESULTS

A performance evaluation of a 6.02-inch-tip-diameter radial-inflow turbine was made by observing its operation in argon over a range of inlet total pressures from 1.2 to 9.4 pounds per square inch absolute. The corresponding Reynolds numbers ranged from 20 000 to 225 000. The results of this evaluation are summarized as follows:

1. In terms of total efficiency, the turbine performance, while operating at equivalent design speed and pressure ratio, increased from about 0.85 to 0.90 as the inlet total pressure was increased from 1.2 to 9.4 pounds per square inch absolute (corresponding Reynolds numbers of 20 000 to 225 000). The corresponding static efficiency increased from 0.80 to 0.84. The turbine equivalent weight flow increased approximately 2 percent over the Reynolds number range. The increase in efficiency and equivalent weight flow thus resulted in a 6 percent increase in equivalent torque over the range investigated.

2. The experimental variation of losses with Reynolds number was in good agreement with an empirical equation in which 70 percent of the losses were the result of skin friction and varied with the Reynolds number to the 1/5 power. The remaining 30 percent of the losses were the result of kinetic losses that are not Reynolds number dependent.

3. The probable errors associated with single determinations of total and static efficiency were 0.009 and 0.008, respectively, for a 16 point data set at design Reynolds number. An increase in probable error occurred as the Reynolds number decreased.

Lewis Research Center,
    National Aeronautics and Space Administration,
        Cleveland, Ohio, November 1, 1966,
            120-27-03-13-22.

APPENDIX A

SYMBOLS

A   stator exit flow area, ft^2

d   deviation from mean value

g   gravitational constant, 32.174 ft/sec^2

H'   isentropic specific work, based on total pressure ratio, ft-lb/lb

h   inlet rotor blade height (see fig. 4), ft

Δh   specific work, Btu/lb

J   mechanical equivalent of heat, 778.029 ft-lb/Btu

N   rotative turbine speed, rpm

N_s specific speed, $NQ^{1/2}/(H')^{3/4}$, $\text{ft}^{3/4}/(\text{min})(\text{sec}^{1/2})$

p   pressure, psia

Δp   orifice pressure drop, psid

Q   volume flow, based on exit conditions, ft^3/sec

Re   Reynolds number, $w/\mu r_t$

r   radius, ft

T   absolute temperature, $^\circ$R

t   stator passage width (see fig. 2), ft

U   blade velocity, ft/sec

V   absolute gas velocity, ft/sec

V_j ideal jet speed corresponding to total- to static-pressure ratio across turbine, $\sqrt{2gJ\,\Delta h_{id}}$, ft/sec

w   weight flow, lb/sec

α absolute gas flow angle, measured from through flow direction, deg

Γ torque, in.-lb

γ ratio of specific heats

δ ratio of inlet total pressure to U.S. standard sea-level pressure, p'/p*

ϵ function of γ used to convert parameters at test conditions to air equivalent conditions at U.S. standard sea-level,

$$\frac{\gamma^*}{\gamma}\left[\frac{\left(\frac{\gamma+1}{2}\right)^{\gamma/(\gamma-1)}}{\left(\frac{\gamma^*+1}{2}\right)^{\gamma^*/(\gamma^*-1)}}\right]$$

η turbine efficiency

η_s static efficiency (based on total- to static-pressure ratio across turbine)

η_T total efficiency (based on total- to total-pressure ratio across turbine)

θ_{cr} squared ratio of critical velocity at turbine inlet to critical velocity at U.S. standard sea-level temperature, $(V_{cr}/V_{cr}^*)^2$

μ gas viscosity, lb/(ft)(sec)

ν blade-jet speed ratio (based on rotor inlet tip speed), U_t/V_j

ρ gas density, lb/ft^3

Subscripts:

cr   conditions corresponding to Mach 1

eq   air equivalent (U.S. standard sea-level conditions)

h   hub

id   ideal

r   radial component

ref   reference value, (Reynolds number, 225 000)

t   tip

1   station at orifice inlet

2   station at turbine inlet (see fig. 4)

3   station at turbine exit (see fig. 4)

Superscripts:

'   absolute total state

*   U.S. standard sea-level conditions (temperature equal to 518.67° R, pressure equal to 14.696 psia)

APPENDIX B

COMPARISON OF REYNOLDS NUMBER EXPRESSIONS

In the section TURBINE REYNOLDS NUMBER it was indicated that the Reynolds number used in this report could be obtained by dropping the factor $1/(2\pi \cos \alpha)$ from the conventional form. For the particular turbine used herein, $\alpha = 72^{\circ}$ at the stator exit so that $2\pi \cos \alpha$ is approximately 2. Therefore, the Reynolds number used in this report is about twice the conventional Reynolds number that would be calculated by making use of the blade height and the stator-exit velocity.

A passage Reynolds number may also be calculated on the basis of the stator passage dimensions. A hydraulic mean diameter, defined as four times the cross-sectional area divided by the perimeter, may be used as the characteristic length. With h as the axial length of the passage and t as its width, the characteristic length is $2ht/(h + t)$. Using

$$w = \rho A V_r = \rho 2\pi r_t h V \cos \alpha$$

where α is the exit angle of the stator and A is the flow area, gives the expression for Reynolds number in the form

$$\text{Reynolds number} = \left(\frac{2t}{h + t}\right)\left(\frac{1}{2\pi \cos \alpha}\right)\left(\frac{w}{\mu r_t}\right)$$

For the turbine used herein, $2\pi \cos \alpha$ is approximately 2 and h is approximately 2t, so that

$$\text{Reynolds number} \cong \frac{1}{3}\frac{w}{\mu r_t}$$

The Reynolds number used is, therefore, about 3 times the passage Reynolds number. Thus, for a particular turbine, or for geometrically similar turbines, the various forms of Reynolds number differ only by a constant factor, and conversions from one form to another can be made easily.

APPENDIX C

CALCULATION OF PROBABLE ERRORS

Calculations were made to determine the probable errors in the measured variables for a 16 point set of data taken at a single set of operating conditions. The probable errors thus determined were used to calculate the probable errors in the calculated quantities, such as weight flow and efficiency. The methods used were standard procedures, such as those discussed in references 5 and 6. Details of the method follow. For each measured variable, consisting of the speed and the various temperatures and pressures measured throughout the turbine, the probable error in a single value was calculated from the following equation:

$$\text{Probable error} = 0.6745\sqrt{\frac{d_1^2 + d_2^2 + \ldots + d_n^2}{n - 1}}$$

where d denotes the deviation of a single value from the mean, and n is the number of readings, sixteen in this case. These values are listed in table II (p. 12).

Using the probable errors in the measured variables, the probable errors in the calculated variables, such as efficiency, were calculated as follows: Let M be a magnitude that is to be calculated

$$M = f(x, y, z)$$

where x, y, z are measured variables having probable errors denoted by dx, dy, and dz, respectively. The probable error in M is given by

$$dM = \sqrt{(dM)_x^2 + (dM)_y^2 + (dM)_z^2}$$

with

$$(dM)_x = \frac{\partial M}{\partial x}dx$$

$$(dM)_y = \frac{\partial M}{\partial y}dy$$

$$(dM)_z = \frac{\partial M}{\partial z}dz$$

The probable errors in measured variables, as shown in table II, were used in the preceding equations to calculate probable errors in efficiencies and in weight flow. The same probable errors in measured variables were used to calculate probable errors in efficiency and in weight flow for each of the six Reynolds numbers reported. The results are shown in figures 11 and 12.

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